TABLE I
COMPARISON OF THE EFFECTIVE DIELECTRIC CONSTANT ϵ_{reff} BETWEEN THE PREVIOUS AND THE PRESENT METHOD PB ($\epsilon_r = 8, w/h = 1, \mu_r = 1.0$)

METHOD		h/λ_0 0.005	h/λ_0 0.05	h/λ_0 0.1	h/λ_0 0.3	h/λ_0 0.7	h/λ_0 1.0
[3]		5.468	6.124	6.742	7.620	7.898	7.945
[6]		5.471	6.130	6.753	7.654	7.914	7.948
[4]	M=4	5.4678	6.1272	6.7576	7.6591	7.9164	7.9562
PB	α_u 7	5.4753	6.1313	6.7560	7.6513	7.9125	7.9539
PB	10	5.4753	6.1315	6.7570	7.6543	7.9141	7.9546
PB	20	5.4753	6.1316	6.7572	7.6550	7.9149	7.9554
PB	30	5.4752	6.1316	6.7572	7.6551	7.9151	7.9556
PB	40	5.4752	6.1316	6.7572	7.6551	7.9151	7.9556
PB: Park-Balanis Method, Basis Function:M=5,N=4							

TABLE II
COMPUTER TIME ON A SUN SPARC STATION FOR THE CALCULATION OF THE EFFECTIVE DIELECTRIC CONSTANT WITH TWO DIFFERENT TECHNIQUES

	SDA without acceleration$^{(a)}$ $\alpha_u = 1000$(rad/mm)	The improved Method$^{(b)}$ $\alpha_u = 30$(rad/mm)	Computational Efficiency $\left(\frac{a}{b}\right)$
$\frac{w}{h} = 1$ (ϵ_{reff})	150.12 seconds (6.7563)	5.82 seconds (6.7572)	25.8
$\frac{w}{h} = 0.1$ (ϵ_{reff})	232.060 seconds (5.7728)	4.78 seconds (5.7697)	48.5

integration in the evaluation of the impedance matrix elements was performed by using Gaussian quadrature. The interval of numerical integration is subdivided into small intervals. Gaussian integration is used over each subinterval. The authors found that the number of basis functions $M = 5$, $N = 4$ is sufficient to accurately represent the surface current density in the entire range from $h/\lambda_0 = 0$ to $h/\lambda_0 = 1$.

Table II illustrates a comparison of the computation time between the conventional SDA without asymptotic extraction technique and the proposed method for the calculations of effective dielectric constant for $w/h = 1$ and $w/h = 0.1$ ($h/\lambda_0 = 0.1$). For both techniques, the quasi-TEM [12, p. 450] effective dielectric constant was used as the initial value. Starting with this initial trial solution, the results shown in Table II converge with an accuracy of 10^{-4}, after the seventh iteration for $w/h = 1$ and after the sixth iteration for $w/h = 0.1$. The integration of the impedance matrix elements in the conventional SDA requires truncation at a high value of the upper-limit α_u to provide sufficient accuracy, which results in a significantly greater amount of computer time than the proposed method. As shown in Table II, the improved method reduces the computational time by 26 times (for $w/h = 1$) and 49 times (for $w/h = 0.1$) than the conventional SDA.

IV. CONCLUSION

In this paper, the authors have shown that a rigorous full-wave spectral-domain approach using the closed-form asymptotic extraction technique (with choice of Chebyshev basis functions) results in accurate results and significant savings in computation time over the conventional SDA. By using the accurate numerical evaluation of the finite integral and the closed-form asymptotic extraction formula, the computational efficiency has been increased while the results retain their accuracy. It should be emphasized that the closed-form asymptotic formula obtained in this paper can also be applied to multilayer microstrip lines and slotlines.

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Characteristics of Asymmetrical Coupled Lines of a Conductor-Backed Coplanar Type

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***Abstract*— This paper presents for the first time a computer-aided design-oriented (CAD) analytical formula for the determination of the characteristic parameters of asymmetrical coupled lines of a conductor-backed coplanar type. Closed-form expressions are developed for evaluating the self and mutual static capacitances based on a sequence of conformal transformations. The derived formulas show excellent accuracy compared to the results produced by a spectral-domain approach.**

***Index Terms*—Asymmetrical coupled lines, coplanar waveguide.**

I. INTRODUCTION

Coupled transmission lines are used extensively in filters, impedance matching networks, and directional couplers. The main advantages of asymmetrical coupled lines [1] is that they offer added

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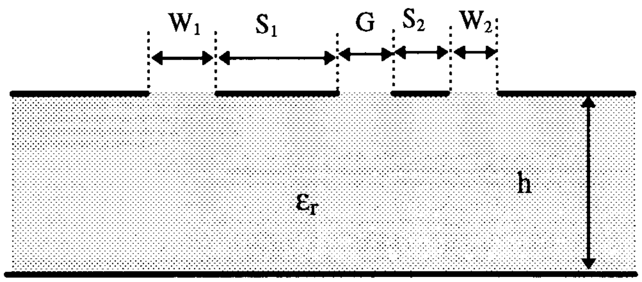

Fig. 1. Asymmetrical coplanar coupled lines.

flexibility in the design of circuits through an additional variable and inherent impedance transforming capability. Various approaches have been reported on the characterization of asymmetrical coupled lines such as the network analog method [2] for microstrip lines, the variational formulation [3] for coplanar-type transmission lines, and the capacitance dividing method [4] for coplanar strips and striplines, as well as microstrip lines. Recently, the coplanar waveguide has received much attention due to its simplicity, better performance at the millimetric frequencies, and lower fabrication cost. For monolithic microwave integrated circuit (MMIC) applications, where the substrate is typically thin and fragile, a back face metallization is usually employed to increase the mechanical strength as well as to improve heat dissipation. The standard coplanar waveguide (CPW) plus this additional ground plane is often called a conductor-backed CPW. In this paper, a new and accurate closed-form formula is derived for calculating the self and mutual static capacitances of asymmetrical coupled lines of a conductor-backed coplanar type. Numerical results generated by a spectral-domain approach are also included for accuracy verification of the proposed method.

II. Method of Analysis

In an asymmetrical coupled-line structure there are two fundamental modes of propagation, namely the c and π modes. The propagation characteristics [1] can be expressed in terms of two propagation constants, β_c, β_π, and four characteristic impedances, $Z_{i,c}, Z_{i,\pi}$ ($i = 1, 2$). It has also been shown [2] that these parameters can be expressed in terms of the capacitance and inductance matrices. Furthermore, the self- and mutual-inductance values are evaluated from the capacitance matrix of the same geometry in air. Note that the capacitance matrix is defined [3] as

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} C_1 & -C_m \\ -C_m & C_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{1}$$

where V_1 and Q_1 are the potential and total charges on the left strip, V_2, and Q_2 are those on the right strip, and C_1, C_2, and C_m are the self and mutual capacitances.

The structure to be studied is shown in Fig. 1, where all the conductors are assumed to be infinitely thin and perfectly conducting. It is also assumed that the air-dielectric interfaces (where all the conductors are located) can be dealt with as though perfect magnetic walls are present in them. Hence, the total self and mutual capacitances can be considered as the sum of the partial self and mutual capacitances in the air region and in the dielectric substrate. To determine the self and mutual capacitances in the air region a sequence of conformal transformations is used as depicted in Fig. 2. The lower half-plane in the t domain is mapped onto the w domain through

$$w = \int_{t_a}^{t} \frac{dt}{\sqrt{(t-t_a)(t-t_b)(t-t_c)(t-t_d)}} \tag{2}$$

and the dimensional parameters o, p, and q are given by

$$o = \frac{W}{H} = \frac{K(k)}{K'(k)} \tag{3a}$$

$$k = \sqrt{\frac{DS_1}{(S_1+G)(S_1+W_1)}} \tag{3b}$$

$$D = W_1 + S_1 + G \tag{3c}$$

$$p = \frac{P}{W} \frac{F\left[\arcsin\sqrt{\frac{(D+S_2+W_2)(S_1+G)}{D(S_1+G+S_2+W_2)}},\, k\right]}{K(k)} \tag{3d}$$

$$q = \frac{Q}{W} \frac{F\left[\arcsin\sqrt{\frac{(D+S_2)(S_1+G)}{D(G+S_1+S_2)}},\, k\right]}{K(k)} \tag{3e}$$

where $K(k)$ and $K'(k)$ are the complete elliptic integral of the first kind and its complement, and $F(\phi, k)$ is the incomplete elliptic integral of the first kind, written in Jacobi's notation. The values of C_1 and C_m are obtained by setting $V_1 = 1$ and $V_2 = 0$, and by placing a magnetic wall at the center of the slot ($2R = P + Q$) as indicated in Fig. 2. The configuration in the w domain can then be considered as two isolated capacitances [5] and the partial self and mutual capacitances for the air region are, therefore, given by

$$C_1 = \varepsilon_0 \left\{ \frac{K(\alpha\beta)}{K'(\alpha\beta)} + \frac{K(\gamma\sigma)}{K'(\gamma\sigma)} \right\} \tag{4a}$$

$$C_m = \varepsilon_0 \frac{K(\gamma\sigma)}{K'(\gamma\sigma)} \tag{4b}$$

where

$$\frac{K(\alpha)}{K'(\alpha)} = or \tag{5a}$$

$$\frac{F(\arcsin\beta,\, \alpha)}{K(\alpha)} = \frac{p}{r} \tag{5b}$$

$$\frac{K(\gamma)}{K'(\gamma)} = o(1-r) \tag{5c}$$

$$\frac{F(\arcsin\sigma,\, \gamma)}{K(\gamma)} = \frac{1-q}{1-r} \tag{5d}$$

and $r = R/W$. Simple and accurate formulas are available [6] for solving (3)–(5). Similarly, C_2 can be determined from the same set of expressions derived in (3)–(5) but with C_1, S_1, S_2, W_1, and W_2 replaced by C_2, S_2, S_1, W_2, and W_1, respectively.

On the other hand, the partial self and mutual capacitances for the dielectric region are computed by the conformal transformation as shown in Fig. 3. Note that the dielectric region in the z domain is transformed onto the lower half-plane in the t domain through the mapping

$$t = e^{z\pi/h}. \tag{6}$$

Hence, the values of C_1, C_2, and C_m for the dielectric region can be found using the procedures described previously, but with S_1, S_2, G, W_1, W_2, and ε_o replaced by S_1', S_2', G', W_1', W_2', and $\varepsilon_o\varepsilon_r$, respectively, where

$$W_1' = e^{W_1\pi/h} - 1 \tag{7a}$$

$$S_1' = e^{W_1\pi/h}(e^{S_1\pi/h} - 1) \tag{7b}$$

$$G' = e^{(W_1+S_1)\pi/h}(e^{G\pi/h} - 1) \tag{7c}$$

$$S_2' = e^{(W_1+S_1+G)\pi/h}(e^{S_2\pi/h} - 1) \tag{7d}$$

$$W_2' = e^{(W_1+S_1+G+S_2)\pi/h}(e^{W_2\pi/h} - 1). \tag{7e}$$

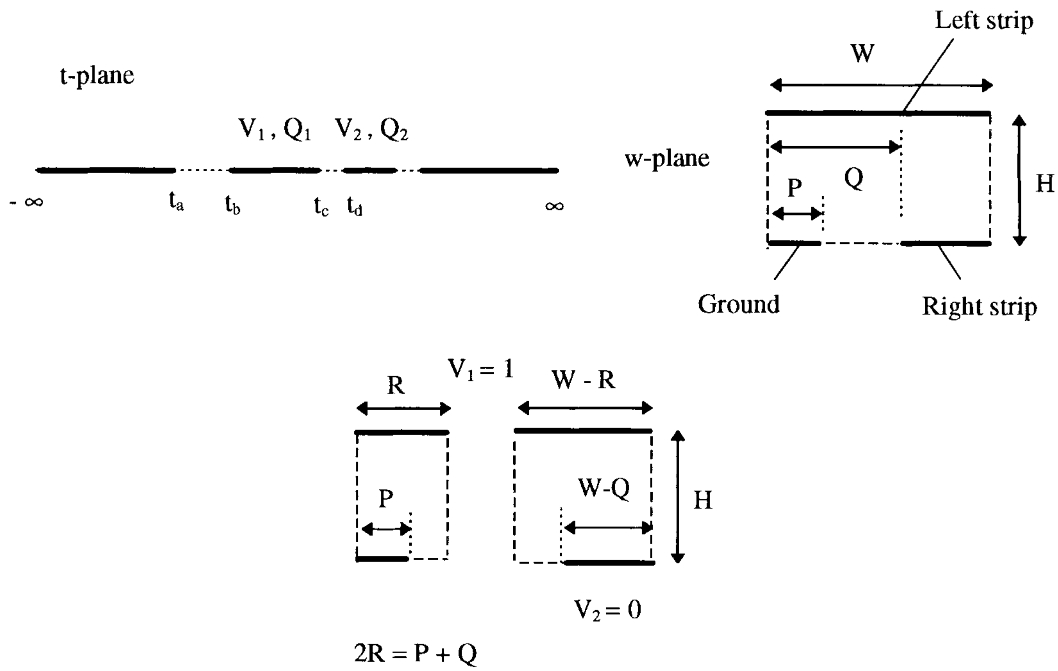

Fig. 2. Capacitance evaluation by conformal transformations.

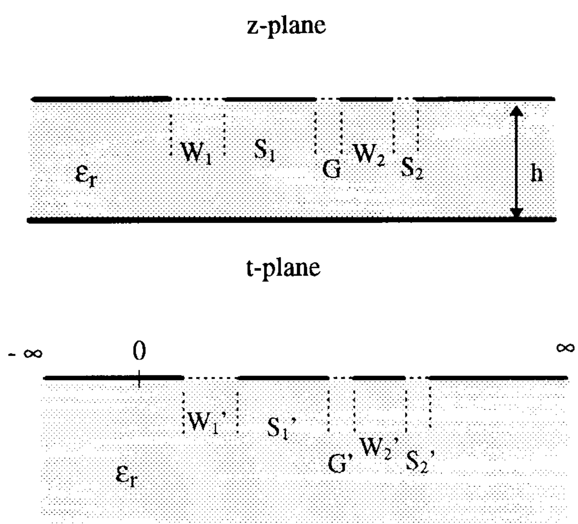

Fig. 3. Conformal mapping for the dielectric region.

TABLE I
SELF AND MUTUAL CAPACITANCE VALUES EVALUATED BY THE PROPOSED METHOD AND A SPECTRAL-DOMAIN APPROACH ($\varepsilon_r = 12.9, W_1 = W_2 = S_2$)

S_2/h	G/S_2	This paper $C_1 / C_2 / C_m$ (pF) $S_1/S_2 = 0.5$ & 2	Spectral Domain [7] $C_1 / C_2 / C_m$ (pF) $S_1/S_2 = 0.5$ & 2
0.2	0.2	170.4 / 205.6 / 92.4 252.0 / 206.9 / 110.0	171.2 / 206.2 / 93.0 252.6 / 207.6 / 110.4
0.2	0.5	145.3 / 177.5 / 61.9 221.3 / 178.5 / 77.0	146.1 / 178.2 / 62.3 222.0 / 179.2 / 77.3
0.2	1.0	131.4 / 160.9 / 41.8 201.9 / 161.5 / 53.9	132.2 / 161.6 / 42.1 202.5 / 161.9 / 54.0
1.0	0.2	192.8 / 257.5 / 70.9 376.1 / 257.8 / 75.5	193.9 / 258.7 / 70.8 377.3 / 258.8 / 75.3
1.0	0.5	173.2 / 236.6 / 38.2 354.6 / 236.7 / 41.5	174.3 / 237.8 / 38.2 355.9 / 237.9 / 41.3
1.0	1.0	165.9 / 228.6 / 17.6 346.2 / 228.7 / 19.4	167.4 / 230.1 / 17.4 348.0 / 230.2 / 19.2

III. NUMERICAL RESULTS AND DISCUSSIONS

Table I shows the values of C_1, C_2, and C_m of an asymmetrical coupled lines ($\varepsilon_r = 12.9$) evaluated by the proposed method and by a spectral-domain technique [7]. For the spectral-domain method employed here, a large number of basis functions are used to approximate the potential distribution at the air–dielectric interfaces in obtaining accurate results for the asymmetrical case. Note that the discrepancies between the values obtained by the two approaches shown in Table I are small (<1.5%), for a wide range of S_2/h, S_1/S_2, and G/S_2 values. The method of analysis mentioned in Section II can easily be applied to the case with upper shielding.

IV. CONCLUSION

A closed-form formula has been derived for evaluating the quasi-static characteristics of asymmetrical coupled lines of conductor-backed coplanar type. The numerical accuracy of this formula has been verified by comparing the results with the ones obtained by a spectral-domain method. The results suggested that the proposed formulas are both accurate and easy to implement, thus making it an excellent choice for use in computer-aided design-oriented (CAD) tools.

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